NASA TECHNICAL TRANSLATION NASA TT F-14,390

Page One Title

INFLUENCE OF THE FLEXIBILITY OF THE ROTOR ELEMENTS ON THE DYNAMIC DEFLECTION OF THE ROTOR AS A WHOLE

N. G. Samarov

Translation of "Vliyaniye Gibkosti Elementov Rotora na Dinamicheskiy Progi' Rotora v Tselom,"

Izvestiya Vysshikh Uchebnykh Zavedenniy, Aviatsionnaya Tekhnika, Vol. 14, No. 2, 1971, pp 67-75.

(4")

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 AUGUST 1972

Even Roman Odd

11

INFLUENCE OF THE FLEXIBILITY OF THE ROTOR ELEMENTS ON THE DYNAMIC DEFLECTION OF THE ROTOR AS A WHOLE

N. G. Samarov

ABSTRACT: The qualitative influence of an eccentricity of the rotor elements on the dynamic deflection of the rotor is assessed. It is shown that the dynamic deflection of the axis of a rotor of composite design is influenced not only by an eccentricity of the shaft but also by that of the individual rotor elements, in particular of disks and blades. The deflection caused by the individual elements is shown to vary as a function of the rpm to a greater extent than the deflection caused by the eccentricity of the rotor as a whole. A method of determining the rotor imbalance both along the length and radius (and, consequently, determining the spots where balancing loads should be applied) is proposed.

The causes of dynamic deflection of flexible rotors are analyzed in many works [1, 2, etc.]. The authors of these works examine rotor deflection only as a function of the flexibility of the shaft. At the same time, it is noteworthy that the elastic deflection of rotor components (discs, cams, etc.) in modern machines are so great as to affect the magnitude of dynamic deflection of the rotor as a whole. /67*

An attempt is made in this article to evaluate the qualitative effect of the eccentricity of the rotor components on the dynamic deflection of the entire rotor and thereby broaden the range of known causes of increased vibrations.

We will detremine the dynamic deflection of the rotor from a second-order Lagrangian equation:

$$\frac{d}{dt}\frac{\delta T}{\delta \dot{x}} - \frac{\delta T}{\delta x} = -\frac{\delta P}{\delta x} + R. \tag{1}$$

here $T = \frac{m\dot{x}^2}{2}$ is the kinetic energy,

$P = \frac{cx^2}{2}$ is the potential energy,

$R = m\omega^2\rho \sin \omega t$ is the centrifugal force of inertia

c is the rotor rigidity

m is the rotor mass,

* Numbers in the margin indicate foreign pagination.

ω is the angular velocity of rotation,

ρ is the rotor eccentricity

x is the displacement of the center of gravity of the rotor in the presence of vibrations.

To solve (1) we will assume

$$x = A \sin\omega t,$$

where A is the maximum amplitude of the vibration of the center of gravity of the rotor,

$$\ddot{x} = -A\omega^2 \sin\omega t.$$

Substituting the kinetic and potential energies in (1) we will obtain $m\ddot{x} = -cx + R$. Substituting x for A $\sin\omega t$ we will obtain

$$-mA\omega^2 \sin\omega t + cA \sin\omega t = m\omega^2\rho \sin\omega t,$$

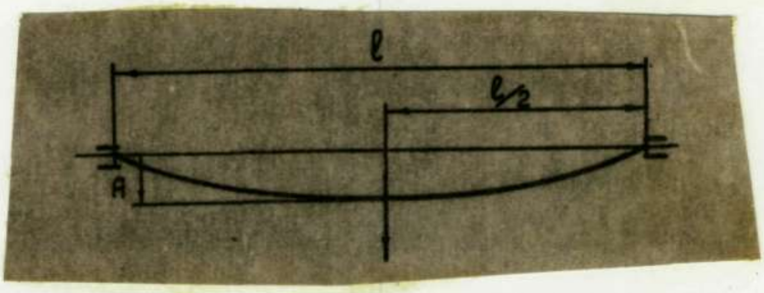
hence /68

$$A = \frac{m\omega^2\rho}{c - m\omega^2} = \frac{\frac{m}{c}\omega^2\rho}{1 - \frac{m}{c}\omega^2}.$$

Considering that $\frac{c}{m} = \omega^2_{cr}$, we will have :

$$A = \frac{\frac{\omega^2}{\omega^2_{cr}}}{1 - \frac{\omega^2}{\omega^2_{cr}}}\rho. \quad (2)$$

This is the known dynamic deflection of the rotor when examining a single-disc design on a weightless shaft (see the figure).



In this expression

$\frac{\omega^2}{\omega^2_{cr}}\rho$ is the static component of deflection

$\beta = \frac{1}{1-\frac{\omega^2}{\omega^2_{cr}}}$ is the dynamic component or coefficient of amplification of the oscillations, depending on the proximity to resonance.

In the derivation of (2) the condition that ρ is a constant retaining its value over the entire range of rotor rpm was observed.

Recalling, however, the flexibility of rotor components, especially the discs, displacement of the center of gravity of the rotor relative to the axis of rotation will not be constant in this case. Under the given conditions the initial displacement of the rotor center of gravity will change as a function of the eccentricity of the rotor components, their rigidity, mass, and finally, rpm.

In order to take into account the variability of the eccentricity of the rotor as a whole, we shall assume that its initial imbalance is absent, $r_{initial} = 0$.

During rotation, elastic eccentricity develops in the rotor as a consequence of the elastic deformation of its components. It should be noted that rotor discs often have a large initial imbalance as a result of asymmetry or eccentricity of the mounting of the disc on the shaft. Despite the fact that the disc is balanced, discrepancy between the radii of the initial imbalance and the radius of installation of the counterbalance leads to elastic deformations of the disc. Let m_1, ρ_1, /69 and c_1 represent the mass, initial eccentricity, and rigidity of the disc, respectively. In this case the elastic displacement of the center of gravity of the disc (A_1) will be

$$A_1 = \frac{m_1 \omega^2 \rho_1}{c_1}. \qquad (3)$$

Considering that we are discussing a rotor with one disc on a weightless shaft

$$A_1 = \rho_{el},$$

where ρ_{el} is the elastic displacement of the center of gravity of the rotor as a whole. Substituting (3) into (2) for the dynamic deflection of the rotor we obtain

$$A = \frac{\frac{\omega^2}{\omega_{cr}^2} \cdot \frac{m_1}{c_1} \omega^2 \rho_1}{1 - \frac{\omega^2}{\omega_{cr}^2}}. \tag{4}$$

Here, $m_1 \rho_1$, and c_1 are constants. We shall replace them with analogous values that determine deflection of the rotor as a whole:

$$\frac{m_1}{c_1} \rho_1 = \frac{m}{c} \rho_{1\,(red)}, \tag{5}$$

where $\rho_{1\,(red)}$ is the initial eccentricity of the disc, reduced to the elastic parameters of the rotor as a whole. We shall call this "second-order imbalance".

Equation (5) in turn can be valid only in those cases when $\rho_1 \gg \rho_{(red)}$ since $c_1 \gg c$. Conditions under which the eccentricity of the disc is one order or magnitude greater than that of the rotor as a whole are very frequently encountered in practice. With this in mind, the problem at hand may be noteworthy with regard to various mechanisms.

Substituting (5) into (4) we obtain

$$A_2 = \frac{\frac{\omega^4}{\omega_{cr}^4} \rho_{1\,(red)}}{1 - \frac{\omega^2}{\omega_{cr}^2}}. \tag{6}$$

Thus, dynamic deflection of the rotor is a function of second-order imbalance and hence changes as a function of ω^4, in contrast to ω^2 for the case of first--order imbalance.

It can be shown analogously that rotor deflection depends on third-order imbalance (for instance, on the eccentricity of the disc blades) and changes as a function of ω^6, etc.

In the general case, when there is eccentricity of n components, as well as eccentricity of the rotor as a whole, coinciding both in magnitude and in phase with the eccentricity of the components, dynamic deflection of the rotor is expressed by the following power series:

$$A = A_1 + A_2 + A_3 + \cdots + A_n,$$

$$A = \frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{1 - \frac{\omega^2}{\omega_{cr}^2}} + \frac{\frac{\omega^4}{\omega_{cr}^4}\rho}{1 - \frac{\omega^2}{\omega_{cr}^2}} + \frac{\frac{\omega^6}{\omega_{cr}^6}\rho}{1 - \frac{\omega^2}{\omega_{cr}^2}} + \cdots + \frac{\frac{\omega^{2n}}{\omega_{cr}^{2n}}\rho}{1 - \frac{\omega^2}{\omega_{cr}^2}} = \frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{\left(1 - \frac{\omega^2}{\omega_{cr}^2}\right)^2}. \tag{7}$$

It is obvious from (7) that dynamic deflection of the rotor changes more progressively under the stated conditions than in the case when it is a function of shaft eccentricity alone. /70

In other words, considering the flexibility of all components, the amplification coefficient of the vibrations will have an exponent twice that in (2). The first two terms of series (7) are of dominant importance. This is very clear when $\omega = 0.71\omega_{cr}$.

$$A = \rho + 0{,}5\rho + 0{,}25\rho + 0{,}125\rho + \cdots + = 2\rho. \qquad (7A)$$

The first term of series (7), as was shown, is characterized by imbalance of the rotor as a whole, and the second by the initial imbalance of the discs (second-order imbalance). Consequently, with equal eccentricity the effect of the latter on the regime $\omega = 0.71\omega_{cr}$ comprises 50% of the first-order imbalance relative to the overall level of the vibrations.

As resonance is approached, relative second-order imbalance increases. In the regime $\omega = 0.9\omega_{cr}$ its value reaches 80% in relation to the influence of imbalance of the rotor as a whole. The answer to the question of the cause of the source of vibrations can be found by comparing the dynamic deflection of the rotor in two different regimes.

The following regimes were selected for this purpose:

$$\omega_1 = 0.5\omega_{cr} \text{ and } \omega_2 = 0.71\delta_{cr}.$$

The ratios of calculated amplitudes in these regimes yield whole numbers.

The theoretical and experimental data are presented in Table 1.

We should point out that shaft imperfection cannot be measured on just any machine subjected to vibrations. Most often the amplitude of vibrations of the housings is monitored rather than shaft deflection, especially since housing rigidity is often less than shaft rigidity. In such cases, other regimes and relations for calculating amplitudes may be used for determining the point of concentration of imbalance. /71

TABLE 1.

Exponent with ω	A	A_1/ρ for $\omega_1 = 0{,}5\omega_{cr}$	A_2/ρ for $\omega_2 = 0{,}71\omega_{cr}$	A_2/A_1 (theoretical data)	A_2/A_1 (experimental data)
0	$\dfrac{\rho}{1-\dfrac{\omega^2}{\omega_{cr}^2}}$	$\frac{4}{3}$	2	1,5	1,42
2	$\dfrac{\dfrac{\omega^2}{\omega_{cr}^2}}{1-\dfrac{\omega^2}{\omega_{cr}^2}}\rho$	$\frac{1}{3}$	1	3,0	2,88
4	$\dfrac{\dfrac{\omega^4}{\omega_{cr}^4}}{1-\dfrac{\omega^2}{\omega_{cr}^2}}\rho$	$\frac{1}{12}$	2	6,0	5,92

Note: Commas indicate decimal points.

The principle of diagnosis, however, remains the same: imblance of the rotor as a whole causes vibrations, dependent on ω^2, and imbalance of rotor elements - vibrations depending on angular velocity of a higher degree.

In addition, other relations are possible when vibrations will change with increasing shaft rpm as a function of angular velocity with an exponent of at least 2. This phenomenon is possible when rotor imbalance is so great that the centrifugal force caused by it exceeds the force of the rotor weight. Then the shaft separates from the bearing and begins to wear it down. This is the so--called third mode of bearing operation, first investigated by B. V. Shitikov [3]. During operation in this mode, the rotor experiences additional displacements of the center of gravity relative to the axis of rotation, equal to the radial clearance in the bearing. If this clearance is assumed to be equal to the initial displacement of the center of gravity ($\Delta = \rho$), characterizing rotor imbalance, we shall obtain the following relationship

$$A = \frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}} + \rho = \frac{\rho}{1-\frac{\omega^2}{\omega_{cr}^2}} = \frac{\left(\frac{\omega}{\omega_{cr}}\right)^0 \rho}{1-\frac{\omega^2}{\omega_{cr}^2}}. \qquad (8)$$

In this condition, the ratio of amplitudes in the modes

$$\omega_1 = 0.5\omega_{cr} \text{ and } \omega_2 = 0.71\omega_{cr}$$

is $A_2/A_1 = 1.5$. Considering that the clearance in the bearing is uaually greater than the initial displacement of the center of gravity of the rotor, $1 < A_2/A_1 < 1.5$ (see Table 1).

It is obvious from the Table that the intensity of the rise in amplitude with rpm makes it possible to determine not only the point of concentration of imbalance, but also the regime of operation of the bearing.

The intensity of the increase in amplitude of oscillations (in the given regimes) depends not only on the exponent for ω but also on the exponent of the vibration amplification coefficient. In the case where all rotor components have identical eccentricity, coinciding in phase, the vibration amplification coefficient influences the amplitude of vibration as the square, it is not difficult to show that with a static load ($A = \frac{\omega^2}{\omega_{cr}^2}\rho$) the exponent of the vibration amplification coefficient is equal to zero. In a real rotor, furthermore, the eccentricity of the components will not be identical and may not coincide in phase. Thus, the exponent of the vibration amplification coefficient will not always be equal to unity, as in (2), and may vary (from zero to 2):

$$0 < p < 2.$$

Let us consider how varying this imbalance of rotor components affects the magnitude of the exponent of the vibration amplification coefficient. /72

In the general case (7) acquires the form

$$\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}} + \frac{\frac{\omega^4}{\omega_{cr}^4}\rho_{red(1)}}{1-\frac{\omega^2}{\omega_{cr}^2}} + \frac{\frac{\omega^6}{\omega_{cr}^6}\rho_{red(2)}}{1-\frac{\omega^2}{\omega_{cr}^2}} + \cdots, \qquad (9)$$

where ρ_{red} (1, 2, 3,...) is the initial eccentricity of the rotor component reduced to elastic displacement of the rotor, on the whole, analogous to (5).

Note that reduced displacement of the center of gravity of the rotor components is usually less than that of the rotor as a whole. The smaller the rotor, the fewer the components. This principle is valid for nearly all machines, although as a rule, the fewer the components, the greater the initial displacement of the center of gravity.

We will examine the possible relationships of eccentricity of rotor components and express them as part of the reduced eccentricity of the rotor as a whole.

The data are reduced to the following series;

$$\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{1}{2}\frac{\omega^4}{\omega_{cr}^4}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{3}{8}\frac{\omega^6}{\omega_{cr}^6}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\cdots=\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{\sqrt{\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)^3}},$$

$$\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{\omega^4}{\omega_{cr}^4}\frac{\rho}{3}}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{\omega^6}{\omega_{cr}^6}\frac{2}{9}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\cdots=\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)^{\frac{4}{3}}},$$

$$\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{\omega^4}{\omega_{cr}^4}\frac{\rho}{4}}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{\omega^6}{\omega_{cr}^6}\frac{5}{32}\rho}{1-\frac{\omega^2}{\omega_{cr}^2}}+\cdots=\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)^{\frac{5}{4}}}.$$

As shown by the examples, if the reduced eccentricities of rotor components, arranged in the plane of the center of gravity, decrease successively, the vibration amplification coefficient will have an exponent less than 2 but greater than 1.

In the general case, dynamic deflection of the rotor will change as a function of the vibration amplification coefficient with exponent p:

$$A=\frac{\frac{\omega^2}{\omega_{cr}^2}\rho}{\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)^p}, \tag{10}$$

where $1 < p < 2$.

Analysis of the imbalance of various machines with blades has shown that the reduced displacement of the center of gravity of rotor components usually changes exponentially. In such a case, /73

$$A=\frac{\frac{\omega^2}{\omega_{cr}^2}p}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{\omega^4}{\omega_{cr}^4}\frac{p}{2!}}{1-\frac{\omega^2}{\omega_{cr}^2}}+\frac{\frac{\omega^6}{\omega_{cr}^6}\frac{p}{3!}}{1-\frac{\omega^2}{\omega_{cr}^2}}+\cdots=\frac{\left(e^{\frac{\omega^2}{\omega_{cr}^2}}-1\right)p}{1-\frac{\omega^2}{\omega_{cr}^2}}. \tag{11}$$

It can be shown that the sum of the terms of series (11) after transformation acquires the form

$$A=\frac{\left(e^{\frac{\omega^2}{\omega_{cr}^2}}-1\right)p}{1-\frac{\omega^2}{\omega_{cr}^2}}\approx\frac{\frac{\omega^2}{\omega_{cr}^2}p}{\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)^{\sqrt{2}}}.$$

Let us determine p for the general case of imbalance of the rotor.

Taking the logarithm of both sides of (10) we obtain:

$$p=\frac{\ln\frac{\omega^2}{\omega_{cr}^2}p-\ln A}{\ln\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)}. \tag{12}$$

Substituting into (12) the partial values of A, we find the corresponding values of p.

If $A=\dfrac{\frac{\omega^2}{\omega_{cr}^2}p}{1-\frac{\omega^2}{\omega_{cr}^2}}$, then p = 1. (13)

If $A=\dfrac{\frac{\omega^2}{\omega_{cr}^2}p}{\left(1-\frac{\omega^2}{\omega_{cr}^2}\right)^2}$, то $p=2$. (14)

If $A=\dfrac{\omega^2}{\omega_{cr}^2}p$, то $p=0$. (15)

$$A = \frac{e^{\frac{\omega^2}{\omega_{cr}^2}} - 1}{1 - \frac{\omega^2}{\omega_{cr}^2}} p, \text{ then } p \approx \sqrt{2}. \qquad (16)$$

Analyzing the partial cases of (12) we see that the exponent of the vibration amplification coefficient indicates the point of concentration of imbalance in the rotor, i.e., the cause of vibration.

Thus, if the exponent is zero, the rotor as a whole is out of balance, and the imbalance is concentrated near the rotor bearing.

If the exponent is equal to 1 the rotor as a whole is out of balance and /74
the imbalance is concentrated in a plane passing through the center of gravity of the rotor.

If the exponent is greater than 1, not only is the rotor as a whole out of balance, but so are its components. Knowledge of the plane of concentration of imbalance, in turn, indicates the means of eliminating the vibrations, i.e., how to balance the system. Either the rotor as a whole is balanced in two or three planes of correction or its components are balanced, depending on the point of imbalance.

The exponent of the vibration amplification coefficient can be determined on an operating machine by analyzing its operating characteristics, similar to the way the exponent of ω is determined. The following regimes of operation are convenient for this purpose: $\omega_2 = 0.71\omega_{cr}$ and $\omega_3 = 0.83\omega_{cr}$.

The ratios of rotor deflection amplitude in these regimes yield completely defined dimensionless values which correspond to specific values of the exponent for the vibration amplification coefficient. These regimes are used because they are closer to resonance than those used for determining the exponent of ω.

Modes of operation relatively far from resonance in which the vibration amplification coefficient has less effect than the static factor are convenient for determining the exponent of ω.

Regimes of operation closer to resonance, in which the effect of the dynamic factor is greater than that of the static factor are convenient for determining

Regimes of operation closer to resonance, in which the effect of the dynamic factor is greater than that of the static factor are convenient for determining the exponent for the vibration amplification coefficient.

The regime $\omega_2 = 0.71\omega_{cr}$ is "boundary" since here the static and dynamic /75 factors have their main effect.

The theoretical amplification ratios are presented in Table 2.

TABLE 2.

Exponent of oscillation amplification coefficient p	A	$\frac{A_2}{\rho}$ at $\omega_2 = 0,71\omega_{cr}$	$\frac{A_3}{\rho}$ at $\omega_3 = 0,83\,\omega_{cr}$	$\frac{A_3}{A_2}$
0	$\frac{\omega^2}{\omega^2_{cr}}\rho$	$\frac{1}{2}$	$\frac{3}{4}$	1,5
1	$\frac{\frac{\omega^2}{\omega^2_{cr}}}{1-\frac{\omega^2}{\omega^2_{cr}}}\rho$	1	3	3,0
2	$\frac{\frac{\omega^2}{\omega^2_{cr}}}{\left(1-\frac{\omega^2}{\omega^2_{cr}}\right)^2}\rho$	2	12	6,0
$\sqrt{2}$	$\frac{\frac{\omega^2}{\omega^2_{cr}}\rho}{\left(1-\frac{\omega^2}{\omega^2_{cr}}\right)^{\sqrt{2}}}$	$\sqrt{2}$	6	4,25

Note: Commas indicate decimal points.

Using the procedure presented herein, it is possible to determine the point of concentration of imbalance of a rotor both by length (Table 1) and radius (Table 2) and consequently to determine the points that are most convenient for placing counterweights for the purpose of balancing.

REFERENCES

Page One Title

1. Hartog, D., "*Mekhanicheskiye Kolebaniya* [Mechanical Vibrations]", "Fizmatgiz" Press, 1960.
2. Timoshenko, S. P., "*Kolebaniyaev Inzheñèrhom Dele* [Vibrations in Engineering]", "Nauka" Press, 1957.
3. Shitikov, B. V., "*Dinamicheskaya Balansirovka Rotorov Turbiny pri Postroyke i Remonte* [Dynamic Balancing of Turbine Rotors During Assembly and Repairs]", "Transzheldorizdat" Press, 1948.

Translated for the National Aeronautics and Space Administration under contract No. NASw-2037 by Techtran Corporation, P. O. Box 729, Glen Burnie, Maryland 21061, translator: Orville E. Humphrey, Jr.

Cover Page Source

(4")

NASA